On joint synchronization of clock offset and skew for Wireless Sensor Networks under exponential delay

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***Abstract*—In this paper, the problem of clock synchronization for Wireless Sensor Network (WSN) under exponential delay is analyzed based on two-way message exchange mechanism. The Maximum Likelihood Estimator (MLE) for joint estimation of the clock offset and clock skew is derived, and an approximate Cramer-Rao Lower Bound (CRLB) is also developed. Simulation results verify that the proposed estimator gives improved performance compared to an existing algorithm.**

I. Introduction

Wireless Sensor Network (WSN), emerged as an important research area in recent years, consists of many small-scale miniature devices (or sensor nodes) capable of onboard sensing, computing and communications. WSNs are used in industrial and commercial applications to monitor data that would be difficult or inconvenient to monitor using wired equipment, such as monitoring the health status of environment, controlling industrial machines and home appliances, fire detection and object tracking, etc. [1] [2]. Most of these applications require collaborative execution of a distributed task amongst a large set of synchronized sensor nodes. Furthermore, data fusion, power management and transmission scheduling require all the nodes running on a common time frame. However, every individual sensor in a WSN has its own clock. Different clocks will drift from each other with time due to many factors, such as imperfection of the oscillators and environmental changes. This makes clock synchronization between different nodes an indispensable piece of infrastructure.

Clock synchronization is not an easy task in practice due to several unique properties of WSN. The first and most important one is the limited power supply in low-end sensor nodes. Due to harsh operating conditions, nodes in WSNs are mostly left unattended for their lifetimes without any maintenance or battery replacement. To save power, each synchronization procedure should be simple and the frequency of re-synchronization should be low. This makes simplicity and accuracy the primary concerns of clock synchronization algorithms for WSNs.

The second challenge of clock synchronization in WSN is the unknown message delays in physical and MAC layers. Kopetz and Ochsenreiter [4] for the first time analyzed the process of message delay and decompose the unknown delay into several components: send time, access time, transmission time, propagation time, reception time and receive time. These delay components can be grouped into two portions: the fixed delay and the random delay. The fixed delay is usually unknown, and if it is not modeled explicitly, it will be treated as a part of time offset, thus lowering the accuracy of timing parameter estimation. On the other hand, the random delay has been modeled as random variables following different distributions (such as Gaussian distribution, exponential distribution, Gamma and Weibull distribution) in the literature based on different justifications and applications, and the difficulty of designing an optimal algorithm for joint estimation of clock offset and clock skew largely depends on the modeling of this random delay.

When the random delay follows Gaussian distribution, the optimal estimator has been given in [7]. However, as pointed out in [3], in many cases, (e.g., when the point-to-point HRX (Hypothetical Reference Connection) topology is of interest), the link delay between two nodes is appropriately represented as a regular M/M/1 queue, and the random delay should be modeled as exponential random variables. In this case, it is much more difficult to design the optimal clock synchronizer. Jeske [5] derived the Maximum Likelihood Estimator (MLE) of clock offset with an unknown fixed delay. But unfortunately the clock skew is not considered, which may result in frequent re-synchronization. Therefore, Noh *et al.* [6] proposed an algorithm for joint estimation of clock offset and clock skew by treating the fixed delay as a nuisance paramter (denoted as EMLLE in this paper). Unfortunately, in EMLLE, not all the available data are used, thus the performance of the EMLLE is limited.

In this paper, the MLE for joint estimation of the clock offset and clock skew is derived when the fixed delay is unknown and the random delay follows exponential distribution. In general, the Cramer-Rao Lower Bound (CRLB) does not exist for exponential delay, but by using a mild approximation, an approximate CRLB is derived. Simulation results confirm that the proposed estimator provides improved performance over the EMLLE.

II. System model

We consider the synchronization between a parent node P and its child node S based on a two-way timing message exchange mechanism as shown in Fig. 1. In the i^{th} round of message exchange, node S sends a synchronization message to

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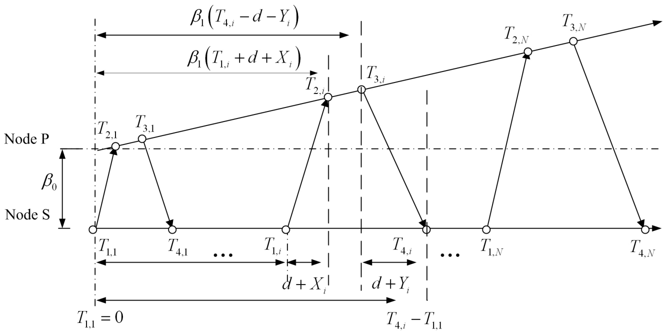

Fig. 1. Two-way time-stamps exchange between two nodes S and P.

node P at $T_{1,i}$. Node P records its time $T_{2,i}$ at the reception of that message, and replies node S at $T_{3,i}$. The replied message contains both time-stamps $T_{2,i}$ and $T_{3,i}$. Then node S records the reception time of node P's reply as $T_{4,i}$. Note that $T_{1,i}$ and $T_{4,i}$ are the time stamps recorded by the clock of node S, while $T_{2,i}$ and $T_{3,i}$ are recorded by that of node P. After N rounds of message exchange, node S obtains a set of time stamps $\{T_{1,i}, T_{2,i}, T_{3,i}, T_{4,i}\}_{i=1}^N$. The above procedure can be modeled as [6]

$$T_{2,i} = \beta_1 \times T_{1,i} + \beta_0 + \beta_1 \times (d + X_i), \tag{1}$$

$$T_{3,i} = \beta_1 \times T_{4,i} + \beta_0 - \beta_1 \times (d + Y_i), \tag{2}$$

where β_0 and β_1 represents the clock offset and clock skew of node S with respect to node P, respectively; d stands for the fixed portion of message delay from one node to another; and X_i and Y_i are variable portions of the message delay. Based on the reasons explained in Section I and the fact that the down/up links between two nodes are usually symmetric, X_i and Y_i are assumed to be independent and identical distributed (i.i.d.) and follow exponential distribution with common rate parameter. The goal is to estimate clock offset β_0 and clock skew β_1 based on the observation of a set of time-stamps $\{T_{1,i}, T_{2,i}, T_{3,i}, T_{4,i}\}_{i=1}^N$.

III. Maximum Likelihood Estimator (MLE) for Clock Offset and Skew

First notice that the fixed delay d is unknown and it is a nuisance parameter in the case of clock synchronization. By observing that the uplink and downlink undergo the same amount of fixed delay, we can rewrite the original model by adding (1) to (2), and we have

$$T_{2,i} + T_{3,i} = \beta_1 \times (T_{1,i} + T_{4,i}) + 2\beta_0 + \beta_1 \times (X_i - Y_i) \ . \tag{3}$$

Dividing the above equation by β_1, defining $\theta_1 = 1/\beta_1$ and $\theta_0 = \beta_0/\beta_1$, and stacking all the time-stamps in matrix form, the model becomes

$$\begin{bmatrix} T_{1,1} + T_{4,1} \\ \vdots \\ T_{1,N} + T_{4,N} \end{bmatrix} = \begin{bmatrix} T_{2,1} + T_{3,1} & -2 \\ \vdots & \vdots \\ T_{2,N} + T_{3,N} & -2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} Y_1 - X_1 \\ \vdots \\ Y_N - X_N \end{bmatrix}. \tag{4}$$

When X_i and Y_i are i.i.d. and follow the exponential distribution with rate parameter λ, it is easy to see that $Z_i \triangleq Y_i - X_i$ follows Lapacian distribution with location parameter 0 and scale parameter $1/\lambda$, which can be represented as $Z_i \sim Laplace(0, 1/\lambda)$. Denote $T_{S,i} = T_{1,i} + T_{4,i}$ and $T_{P,i} = T_{2,i} + T_{3,i}$, the likelihood function can be written as

$$\begin{aligned} &\ln f(\{T_{S,i}, T_{P,i}\}_{i=1}^N; \theta_1, \theta_0) \\ &= N \ln \frac{\lambda}{2} - \lambda \cdot \sum_{i=1}^N \Big| T_{S,i} - \theta_1 T_{P,i} + 2\theta_0 \Big|. \end{aligned} \tag{5}$$

Therefore, the MLE becomes minimizing the second term on the RHS of (5)

$$[\theta_1^*, \theta_0^*] = \arg\min_{\theta_1, \theta_0} \ \sum_{i=1}^N \Big| T_{S,i} - \theta_1 T_{P,i} + 2\theta_0 \Big|, \tag{6}$$

which is an unconstrained L_1-norm minimization problem. Denote the objective function as $F(\theta_1, \theta_0)$, we observe that if θ_1 is given, $F(\theta_0) = 2\sum_{i=1}^N \Big| \theta_0 - 0.5(\theta_1 T_{P,i} - T_{S,i}) \Big|$, and the MLE of θ_0 is the median value of the sequence $\{0.5(\theta_1 T_{P,i} - T_{S,i})\}_{i=1}^N$ [8]. On the other hand, if θ_0 is given, the MLE of θ_1 is the optimal solution of the following problem

$$\arg\min_{\theta_1} \ \sum_{i=1}^N T_{P,i} \Big| \theta_1 - \frac{T_{S,i} + 2\theta_0}{T_{P,i}} \Big|. \tag{7}$$

Treating $(T_{S,i} + 2\theta_0)/T_{P,i}$ as the location of the i^{th} particle with weight $T_{P,i}$ on a horizontal line, it can be proved that the MLE of θ_1 is the so-called weighted median value of the data set $\{[T_{P,i}, (T_{S,i} + 2\theta_0)/T_{P,i}]\}_{i=1}^N$ [9]. The procedure for finding the weighted median value is given as follow.

1) Sort the location sequence $\rho_i \triangleq (T_{S,i} + 2\theta_0)/T_{P,i}$ in ascending order, such that $\rho_{[1]} < \cdots < \rho_{[i]} < \cdots < \rho_{[N]}$, where $\{\rho_{[i]}\}_{i=1}^N$ is the order statistics of $\{\rho_i\}_{i=1}^N$ (that is, $[i]$ is the index of the i^{th} smallest element among $\{\rho_i\}_{i=1}^N$);
2) Find the smallest value of K such that $\sum_{i=[1]}^{[K]} T_{P,i} \geq 0.5 \sum_{i=1}^N T_{P,i}$, and denote the solution as K^*;
3) The weighted median of the data set $\{[T_{P,i}, (T_{S,i} + 2\theta_0)/T_{P,i}]\}_{i=1}^N$ is $\rho_{[K^*]} = (T_{S,[K^*]} + 2\theta_0)/T_{P,[K^*]}$.

Based on the above observations, an iterative algorithm is proposed to find the optimal solution. First, the Least Squares (LS) solution is calculated as

$$[\theta_1^0, \theta_0^0] = \begin{bmatrix} T_{P,1} & -2 \\ \vdots & \vdots \\ T_{P,N} & -2 \end{bmatrix}^\dagger \cdot \begin{bmatrix} T_{S,1} \\ \vdots \\ T_{S,N} \end{bmatrix}, \tag{8}$$

where $[\mathbf{A}]^\dagger$ indicates the pseudo-inverse operation of matrix $\mathbf{A}$. Using the LS solution as the initial estimates of θ_1 and θ_0, the proposed iterative algorithm repeats the following operation until convergence.

1) Estimate θ_1^k as the weighted median value of the sequence $\{[T_{P,i}, (T_{S,i} + 2\theta_0^{k-1})/T_{P,i}]\}_{i=1}^N$;
2) Estimate θ_0^k as the median value of the sequence $0.5\{\theta_1^k T_{P,i} - T_{S,i}\}_{i=1}^N$.

After obtaining the MLE of θ_1 and θ_0, we have $\beta_1^* = 1/\theta_1^*$ and $\beta_0^* = \theta_0^*/\theta_1^*$.

Since both the weighted median and the median operations find the MLE of the corresponding parameter, we have

$$F(\theta_1^{k-1}, \theta_0^{k-1}) \geq F(\theta_1^{k}, \theta_0^{k-1}) \geq F(\theta_1^{k}, \theta_0^{k}). \tag{9}$$

Moreover, the objective function $F(\theta_1, \theta_0)$ in (5) is convex, and there exists no local minimums. Therefore, the proposed algorithm will finally converge to the global optimal solution. And simulation results show that the algorithm usually converges within 10 iterations.

IV. Approximate Cramer-Rao Lower Bound (CRLB)

To find CRLB, we need to take derivative of the likelihood function in (5) with respect to the unknown parameters. But the likelihood function (5) is non-differentiable at $T_{S,i} - \theta_1 T_{P,i} + 2\theta_0 = 0$. Nevertheless, (5) can be approximated with a function that is differentiable. Here we use the following approximation

$$|t| \approx \frac{1}{r} \ln \left[\cosh(rt)\right], \tag{10}$$

where r is a user-defined parameter and can be used to control the accuracy of the approximation. For example, when $r = 200$, the function (10) gives nearly perfect approximation to the absolute value function, as shown in Fig 2.

Since $Z_i = T_{S,i} - \frac{1}{\beta_1} T_{P,i} + \frac{2\beta_0}{\beta_1}$ and follows Laplacian distribution, i.e., $Z_i \sim Laplace(0, 1/\lambda)$, the approximated likelihood function of (5) is given by

$$\begin{aligned} &\ln f(\{T_{P,i}, T_{S,i}\}_{i=1}^N; \beta_1, \beta_0) \\ &\approx N \ln \frac{\lambda}{2} - \lambda \sum_{i=1}^{N} \frac{1}{r} \ln \left\{ \cosh \left[r \left(T_{S,i} - \frac{1}{\beta_1} T_{P,i} + \frac{2\beta_0}{\beta_1} \right) \right] \right\}. \end{aligned} \tag{11}$$

Taking second derivative of the log-likelihood function (11) with respect to β_1 and β_0, we have

$$\begin{aligned} \frac{\partial^2 \ln f}{\partial^2 \beta_1} = &-\lambda \sum (T_{P,i} - 2\beta_0) \left[-2\beta_1^{-3} \tanh(rZ_i) \right. \\ &\left. + r\beta_1^{-4} (T_{P,i} - \beta_0) \mathrm{sech}^2(rZ_i) \right], \end{aligned} \tag{12}$$

$$\frac{\partial^2 \ln f}{\partial^2 \beta_0} = -\lambda \sum \left[4r\beta_1^{-2} \mathrm{sech}^2(rZ_i) \right], \tag{13}$$

$$\begin{aligned} \frac{\partial^2 \ln f}{\partial \beta_1 \partial \beta_0} = &-\lambda \sum \left[-2\beta_1^{-2} \tanh(rZ_i) \right. \\ &\left. + 2r\beta_1^{-3} (T_{P,i} - 2\beta_0) \mathrm{sech}^2(rZ_i) \right]. \end{aligned} \tag{14}$$

To calculate CRLB, we further need to take expectation of (12) – (14) with respect to Z_i. Based on the fact that $\int_0^{+\infty} \tanh(x) e^{-\mu x} \mathrm{d}x = -\int_{-\infty}^{0} \tanh(x) e^{\mu x} \mathrm{d}x$, we have

$$\int_{-\infty}^{+\infty} \tanh(rZ_i) \frac{\lambda}{2} e^{-\lambda Z_i} \mathrm{d}Z_i = 0. \tag{15}$$

Furthermore, we also have

$$\begin{aligned} &\int_{-\infty}^{+\infty} \mathrm{sech}^2(rZ_i) \frac{\lambda}{2} e^{-\lambda Z_i} \mathrm{d}Z_i \\ &= \frac{\lambda}{r} \underbrace{\left\{ \frac{\lambda}{2r} \left[\psi \left(\frac{\lambda + 2r}{4r} \right) - \psi \left(\frac{\lambda}{4r} \right) \right] - 1 \right\}}_{\triangleq V}, \end{aligned} \tag{16}$$

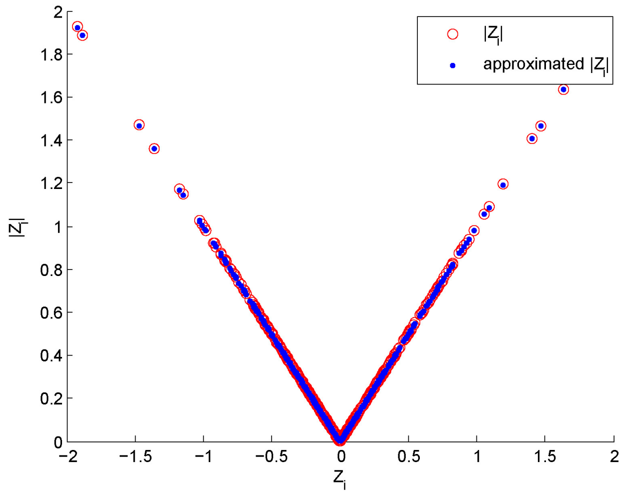

Fig. 2. Approximation of the Laplacian random variable.

because $\int_0^{+\infty} \mathrm{sech}^2(x) e^{-\mu x} \mathrm{d}x = \int_{-\infty}^{0} \mathrm{sech}^2(x) e^{\mu x} \mathrm{d}x = \frac{\mu}{2} \left[\psi \left(\frac{\mu+2}{4} \right) - \psi \left(\frac{\mu}{4} \right) \right] - 1$, where $\psi(x)$ is a special function named Euler psi function and its precise definition can be found in [10, pp.892-896].

Using (15) and (16), it is easy to calculate the expectation of (12) – (14) with respect to Z_i, and the Fisher information matrix is

$$\begin{aligned} \mathrm{FIM}(\beta_1, \beta_0) &= \begin{bmatrix} -\mathbf{E} \frac{\partial^2 \ln f}{\partial \beta_1^2} & -\mathbf{E} \frac{\partial^2 \ln f}{\partial \beta_1 \partial \beta_0} \\ -\mathbf{E} \frac{\partial^2 \ln f}{\partial \beta_0 \partial \beta_1} & -\mathbf{E} \frac{\partial^2 \ln f}{\partial \beta_0^2} \end{bmatrix} \\ &= \frac{\lambda^2}{\beta_1^4} \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{C} \end{bmatrix}, \end{aligned} \tag{17}$$

where $\mathcal{A} \triangleq \sum_{i=1}^{N} V (T_{P,i} - 2\beta_0)^2$, $\mathcal{B} \triangleq 2\beta_1 \sum_{i=1}^{N} V (T_{P,i} - 2\beta_0)$ and $\mathcal{C} \triangleq 4\beta_1^2 N V$. By inverting the matrix (17), it can be shown that the CRLB for each parameter is

$$\mathrm{CRLB}(\beta_1) = \frac{\beta_1^4 \mathcal{C}}{\lambda^2 (\mathcal{AC} - \mathcal{B}^2)}, \tag{18}$$

$$\mathrm{CRLB}(\beta_0) = \frac{\beta_1^4 \mathcal{A}}{\lambda^2 (\mathcal{AC} - \mathcal{B}^2)}. \tag{19}$$

V. Simulation results and discussions

As mentioned in Section I, Noh *et al.* [6] proposed an algorithm EMLLE for joint estimation of the clock offset and clock skew. Here, simulation results are presented to compare the performances of the EMLLE and the proposed estimator. The parameters used in the simulation are $\lambda = 1$, $d = 2$, $\beta_1 = 1.003$ and $\beta_0 = -10$. Each point in the figures is an average of 10000 simulation runs.

Fig. 3 shows the mean squared error (MSE) for estimation of the clock skew β_1 as a function of the number of round of message exchange N. As shown in the figure, the proposed estimator performs much better than EMLLE. The performance of the proposed estimator deviates slightly from the CRLB, which is due to the approximation, and similar behavior is also found in [11]. However, the approximate CRLB can still predict the trend of the performance of the proposed estimator

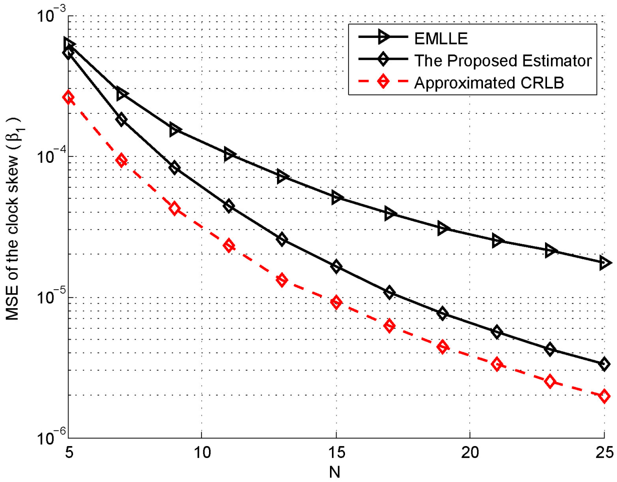

Fig. 3. MSE of estimated clock skew $\hat{\beta}_1$ with respect to the number of rounds of message exchange N.

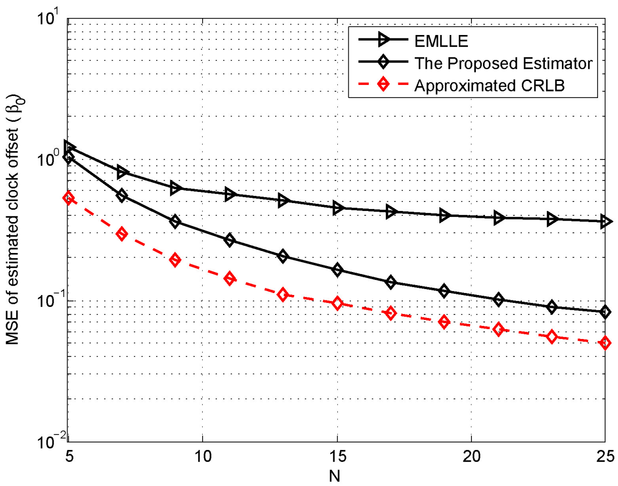

Fig. 4. MSE of estimated clock offset $\hat{\beta}_0$ with respect to the number of rounds of message exchange N.

and serve as the lowest limit. Fig. 4 shows the corresponding results for the clock offset β_0. It can be seen from the figure, the same conclusions as in Fig. 3 can be drawn.

VI. Conclusions

Clock synchronization for WSN in the presence of exponential delay was discussed based on the two-way message exchange mechanism. The MLE for joint estimation of clock skew and clock offset was derived by treating the unknown fixed delay as a nuisance parameter. The CRLB was also derived by approximating the original non-differentiable probability density function with a highly accurate differentiable function. Although the performance of the proposed estimator deviates slightly from CRLB, which seems to be a common result due to the mild approximations [11], the CRLB predicts the trend of the performance very well and gives the lowest limit. The proposed estimator was also compared to an existing algorithm, and simulation results indicate that the proposed estimator gives improved performance compared to the EMLLE.

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